

Air Quality as a Raster Grid

	0	1	2	3	4
0	72	74	80	85	90
1	71	75	82	88	92
2	68	73	85	90	95
3	65	70	80	88	93
4	60	68	78	84	90

Idea:

The city is modeled as a **raster grid** with equally sized cells.

Each cell stores an **Air Quality Index (AQI)** at a spatial location.

For example:

Cell **(2,2)** → AQI **85**

Noise in Spatial Measurements

	0	1	2	3	4
0	72	74	80	85	90
1	71	75	82	88	92
2	68	73	20 ↓	90	95
3	65	70	80	88	93
4	60	68	78	84	90

Problem: Air Quality sensors can produce noisy measurements.

A single cell may contain an AQI much higher or lower than its direct neighbors

For example:

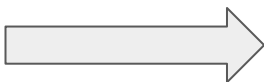
Cell **(2,2)** → AQI **20** while neighbors are much higher.

Question: How to reduce spatial noise?

Solution: Mean Filter (3x3) Smooths the Grid

Idea: Each cell is replaced by the **average** AQI of its local 3x3 neighborhood. For example: cell **(2,2)** is replaced by (sum of neighborhood values) \div 9 = **74**

	0	1	2	3	4
0	72	74	80	85	90
1	71	75	82	88	92
2	68	73	[20]	90	95
3	65	70	80	88	93
4	60	68	78	84	90



	0	1	2	3	4
0	72	74	80	85	90
1	71	75	82	88	92
2	68	73	[74]	90	95
3	65	70	80	88	93
4	60	68	78	84	90

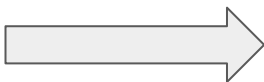
Edge Case: Boundary Cells

Boundary cells don't have complete 3x3 neighborhoods (they have fewer neighboring cells).

Solution: Dynamically shrink the neighborhood (we use only valid cells inside the grid).

Below are two examples highlighted in yellow and green.

	0	1	2	3	4
0	[72]	74	80	85	90
1	71	75	82	88	92
2	68	73	20	90	95
3	65	70	80	88	[93]
4	60	68	78	84	90



	0	1	2	3	4
0	[73]	74	80	85	90
1	71	75	82	88	92
2	68	73	20	90	95
3	65	70	80	88	[90]
4	60	68	78	84	90

Final Smoothed grid

Below is the full transformation after applying the mean filter to the grid (values rounded to the nearest integer). Notice how the noisy value 20 got replaced with a value consistent with its neighbors. The result is a smoothed spatial representation of air quality.

	0	1	2	3	4
0	72	74	80	85	90
1	71	75	82	88	92
2	68	73	20	90	95
3	65	70	80	88	93
4	60	68	78	84	90



	0	1	2	3	4
0	73	76	81	86	89
1	72	68	74	80	90
2	70	67	74	81	91
3	67	65	72	80	90
4	66	70	78	86	89

Gradient Magnitude

A useful transformation we can do to the smoothed grid is computing the gradient magnitude grid. That means computing how quickly the values change as we move across cells. In other words, how quickly air quality changes as we move in space.

73	76	81	86	89
	↑ dy			
72 ← dx	[68]	dx → 74	80	90
	dy ↓			
70	67	74	81	91
67	65	72	80	90
66	70	78	86	89

Steps:

1. **dx**: horizontal change
2. **dy**: vertical change
3. $\text{gradient} = \sqrt{dx^2 + dy^2}$
4. Replace cell with computed gradient.

To the right is the gradient at cell **(1,1)** rounded to 1 decimal point.

73	76	81	86	89
72	[4.6]	74	80	90
70	67	74	81	91
67	65	72	80	90
66	70	78	86	89

Final Gradient Magnitude Grid

	0	1	2	3	4
0	3.2	9.1	5.8	5.2	3.2
1	2.5	4.6	7.1	8.5	4.0
2	3.9	4.6	7.0	8.0	3.2
3	2.8	5.2	7.0	9.1	2.2
4	4.1	6.3	8.1	6.1	3.2

Interpretation:

Relatively large gradient magnitude values indicate steep spatial changes.

Cells **(0,1)** and **(3,3)** have the highest values at 9.1

These locations represent regions where air quality rapidly changes across space.

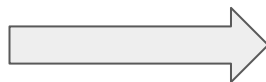
Potential pollution transition zones?

Edge Case: Boundary Cells

Boundary cells used for gradient computation also have fewer neighbors.

Solution: Interior cells use **central differences** (change on both sides horizontally and vertically). At grid boundaries, we instead use **forward** or **backward differences** (change on only the available sides).

73	76	81	86 ← dx	[89]
72	68	74	80	dy ↓ 90
70	67	74	81	91
dy ↑ [67]	dx → 65	72	80	90
dy ↓ 66	70	78	86	89



73	76	81	86	[3.2]
72	68	74	80	90
70	67	74	81	91
[2.8]	65	72	80	90
66	70	78	86	89